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# Numerical investigations of oscillons in 2 dimensions

Mark Hindmarsh<sup>1,\*</sup> and Petja Salmi<sup>1,2,†</sup>

<sup>1</sup>*Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QH, United Kingdom*

<sup>2</sup>*Lorentz Institute of Theoretical Physics, University of Leiden, The Netherlands*

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Oscillons, extremely long-living localized oscillations of a scalar field, are studied in theories with quartic and sine-Gordon potentials in two spatial dimensions. We present qualitative results concentrating largely on a study in frequency space via Fourier analysis of oscillations. Oscillations take place at a fundamental frequency just below the threshold for the production of radiation, with exponentially suppressed harmonics. The time evolution of the oscillation frequency points indirectly to a life time of at least  $10^7$  oscillations. We study also elliptical perturbations of the oscillon, which are shown to decay. We finish by presenting results for boosted and colliding oscillons, which point to a surprising persistence and solitonlike behavior.

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## I. INTRODUCTION

There have been many studies on static nondissipative solutions of classical field theories when the stability of the solutions is guaranteed due to a conservation of a topological charge (see [1,2]) or conserved particle number [3] and there exist examples where exact solutions are known and can be written explicitly in a closed form (see e.g. [4,5]). However, energy can be stored in localized oscillations of a field for a very long time and in this situation there is no obvious conservation law to explain the (meta-)stability of the configuration. In  $1 + 1$  dimensions 'breather' solutions are known and can be written down in closed form [6]. In higher dimensions, the long life time of certain oscillations was pointed out 30 years ago in [7,8] and then rediscovered [9] when the dynamics of first order phase transitions and bubble nucleation was studied. More extended investigations were carried out in [10], where conditions, like the size of the initial bubble, for long-living oscillations were examined. In [8] oscillating objects were called pulsions, we adopt hereafter the definition *oscillon* according to [10].

There is still not a satisfactory explanation for the long life time of oscillons. In [11] it was suggested that the longevity of oscillations could be understood within the framework of adiabatic invariance, i.e. there were a conserved adiabatic charge explaining the metastability of oscillons, called I-balls by the authors [11]. However, this approach assumes negligible gradient energy and non-linearity, i.e. quadratic potential. While the adiabatic invariance, in spite of its restrictions, provides an analytic approach to explain the long, but finite life time, some authors have suggested, based on numerical studies, that the life time could be arbitrarily long [12,13], and even a Lyapunov exponent was suggested to govern the power law of oscillons life time [13].

The possible effects of oscillons on the dynamics of first order phase transitions has been studied e.g. in [14] (for creation of long-living quasilumps from two bubble collisions see [15]). It was pointed out by Riotto [16] that at the electroweak scale thermal fluctuations are too weak to create oscillons and therefore they cannot play a role in first order electroweak phase transition. However, a very recent study [17] exploited the possibility that oscillons would speed up the phase transition in the context of old inflation. It is natural, that oscillons are not persistent in a hot heat bath, but they can be long living in a weak enough thermal environment [18]. Principally, the necessary conditions of potentials to permit oscillons can be fulfilled also in higher dimensional models [19], but on the other hand it should be noticed that stable or extremely long living oscillons (called pseudobreathers by the authors) were obtained in sine-Gordon model only in two spatial dimensions [20].

In addition to [17] there has been recent interest in oscillons and they have been found in some realistic models of great importance. In [21] authors reported that with a particular, experimentally relevant, ratio between Higgs and W boson masses, there exists an oscillon in an SU(2) model that is essentially the bosonic electroweak sector of the standard model, at zero weak mixing angle. An oscillon in the standard model could have significant consequences e.g. for baryogenesis. Another study [22] found oscillon formation after supersymmetric hybrid inflation. The authors worked within the very successful *D*-term inflationary scenario and concluded that for large enough, but fully realistic, couplings in the model, oscillons (called non topological solitons in [22]), form in large numbers and even dominate the energy density in the post-inflationary era (also in [23] energy concentrations, often along the domain walls, were reported in simulations of tachyonic preheating).

Our study here was motivated by creation of oscillons from the collapsing domains in  $\phi^4$  theory and in the sine-

\*Electronic address: [m.b.hindmarsh@sussex.ac.uk](mailto:m.b.hindmarsh@sussex.ac.uk)

†Electronic address: [salmi@lorentz.leidenuniv.nl](mailto:salmi@lorentz.leidenuniv.nl)

Gordon model, a topic we hope to revisit in another study. Here our investigations concentrates on the properties of oscillons when they are created with a Gaussian initial ansatz instead of as a consequence of the evolution after random initial conditions. However, we try to draw the connection by studying the boosted oscillons in the end of this paper. Before that we present results from a stationary setup using Fourier analysis to determine the time evolution of the oscillation frequency when oscillon is created using a spherically symmetric initial ansatz and studies of the collapse to spherical symmetry for an elliptic ansatz.

## II. THE MODELS AND NUMERICAL SETUP

The Lagrangian of a model for a single real scalar field  $\phi$  in the presence of a potential  $V$  is given by

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \quad (1)$$

and the equation of motion thus reads

$$\ddot{\phi} - \nabla^2 \phi + V'(\phi) = 0. \quad (2)$$

We study here two potentials with a discrete symmetry: degenerate double-well quartic potential

$$V(\phi) = \frac{1}{4} \lambda (\phi^2 - v^2)^2 \quad (3)$$

and sine-Gordon potential

$$V(\phi) = \frac{\Lambda^4}{\alpha^2} (1 - \cos(\alpha \phi)). \quad (4)$$

Scaling out the vacuum expectation value and couplings these can be written

$$V(\phi) = \frac{1}{4} (\phi^2 - 1)^2 \quad (5)$$

and

$$V(\phi) = \frac{1}{\pi^2} (1 + \cos(\pi \phi)) \quad (6)$$

so that both (5) and (6) have minima at  $\phi = \pm 1$  and a local maximum at  $\phi = 0$ .

The field equation is evolved on a two-dimensional lattice with periodic boundary conditions using a leapfrog update and a three-point spatial Laplacian accurate to  $O(dx^2)$ . The lattice spacing for the data shown is set to be  $dx = 0.25$  and the time step  $dt = 0.05$ . With that choice the fluctuations in total energy on the lattice are less than 0.2% over  $9 \times 10^7$  iteration steps. We tested our method by reducing both  $dx$  and  $dt$  from the above mentioned choice without observing significant difference in the quantities of main interest here. Unless otherwise stated the simulations for the data shown were carried out in  $800^2$  lattices. We choose periodic boundary conditions rather than absorbing ones because we wish to explore the stability of the oscillons to the small perturbations from any radiation emitted, and because we wish to allow them to move without hitting any boundaries.

## III. PROPERTIES

We start with a Gaussian ansatz

$$\phi(r) = v(1 - A \exp(-r^2/r_0^2)), \quad (7)$$

where  $r$  is the distance to the center of an oscillon  $r = (x_1^2 + x_2^2)^{1/2}$  and the width of the distribution was set to be  $r_0 \simeq 2.9$  (in units of  $(\sqrt{\lambda} v)^{-1}$ ), suggested an optimal choice for a long-living oscillon in [10]. Earlier studies have established the sensitivity both to the spatial size  $r_0$  and the amplitude of the deviation from vacuum. For the data shown amplitude  $A = 1$  so that the center of the oscillon is located in the local maximum of the potential. This choice sets the initial deviation from vacuum to be drastic and ensures that we observe the nonlinear features of the theory and not a small, linear perturbation around the vacuum.

It should be immediately noted that the Gaussian ansatz does not provide the right description of radiation far from oscillon core. This can be seen by studying a small, radially symmetric spatial perturbation  $\varphi$  around the vacuum. For quartic potential (3) substitution  $\phi = v - \varphi(r)$  into (2) leads in lowest order to

$$\frac{d^2 \varphi(r)}{dr^2} + \frac{1}{r} \frac{d\varphi(r)}{dr} + m^2 \varphi(r) = 0, \quad (8)$$

where  $m^2 = 2\lambda v^2$ . This Bessel equation has solution  $\varphi = C_1 \cdot J_0(mr) + C_2 \cdot Y_0(mr)$ , where  $J_\nu$  and  $Y_\nu$  are the Bessel functions of the first and second kind, respectively. Both are oscillatory with an amplitude decaying asymptotically as  $r^{-1/2}$ , thus much more slowly than (7). Repeating the calculation for sine-Gordon potential (4) yields the same result with  $m = \Lambda^2$ . The parameter  $m$  defines a mass in the theory as  $m^2 = V''(\phi)$ , where  $\phi$  is at the minimum of the potential.

Watkins suggested in [12] the following spherically symmetric periodic trial solution (see also [13])

$$\phi(r, t) = \sum_{n=0}^{\infty} f_n(r) \cos(n\omega t), \quad (9)$$

with  $\omega$  a free parameter. Studying the coupled ODE's for  $f_n$  it was noticed that  $f_n \rightarrow 0$  fast as  $n$  increases, so that for  $n \geq 5$  they are negligible. It was also found that solutions existed only for  $\omega < m$ , going some way to explaining the stability: the fundamental mode of oscillation is not quite fast enough to excite outgoing modes. We bear this in mind when discussing our numerical results for  $\phi^4$  and sine-Gordon potentials.

### A. $\phi^4$ potential

The shape of the oscillon profile in  $\phi^4$  theory is shown in Fig. 1. Deviation from the Gaussian form is clearly visible, but not drastic. Figure 2 illustrates the broadening of the

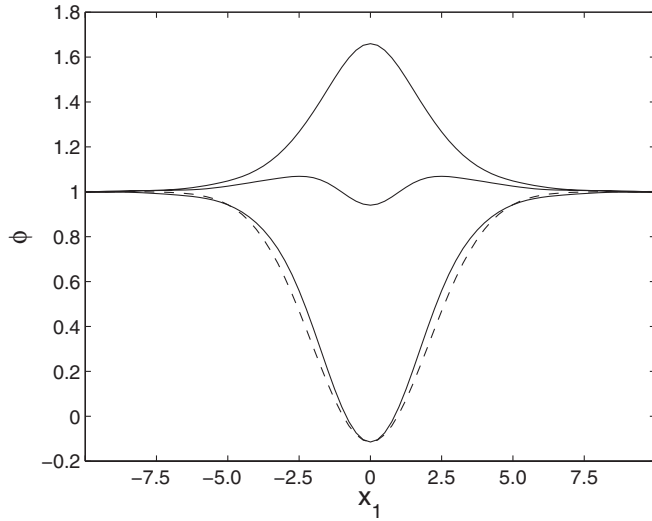


FIG. 1. Oscillon profile at the extrema (above and below the vacuum expectation value) and at crossing the minimum of the potential in the 2D  $\phi^4$  theory. Dashed line shows a Gaussian form with the same amplitude and the width  $r_0$  of the initial profile.

energy density  $\rho(\mathbf{x})$  in the extremum and contraction to a highly concentrated spike as oscillation crosses the minima and kinetic energy dominates the density. The value of the field at the center of the oscillon as a function of time is plotted in the Fig. 3. An oscillation over one period is not symmetric, as the field makes larger excursion and changes more slowly where the potential is flatter whereas it changes rapidly in the region of steeper potential. Variation of the amplitude does not become apparent over this short period of time.

The total energy and the energy inside shells of radius  $r = 0.5r_0$ ,  $1.5r_0$  and  $2.5r_0$  around the center of the oscillon over the complete simulation time ( $4.5 \times 10^6$ ) are shown in Fig. 4. Size  $r = 0.5r_0$  shows large fluctuations depending on the phase of the oscillation, which can be well understood on the basis of the behavior of  $\rho(\mathbf{x})$  over a period (Fig. 2). On the contrary shell  $r = 2.5r_0$  shows relatively thin line and provides a good estimator for the energy of an oscillon. This is further confirmed by comparing the energy inside shells of radius  $r = 2.5r_0$  and  $5r_0$ . They track each other well as can be seen in Fig. 5, where they are plotted for a shorter time interval from the beginning of the simulation. Figure 5 shows also clearly a period in the beginning when oscillon radiates energy rapidly. After that the rate of energy loss is much weaker and the data suggests some evidence for the existence of flat plateaus where energy stays at constant value and abrupt drops between them. These plateaus could be interpreted as series of metastable states (see also [12]). In the end of the simulation more than 70% of the energy is still localized in area covering less than 0.5% of the lattice.

The power spectrum of oscillations was studied by performing Fourier transforms of  $\phi(t, 0)$  in consecutive

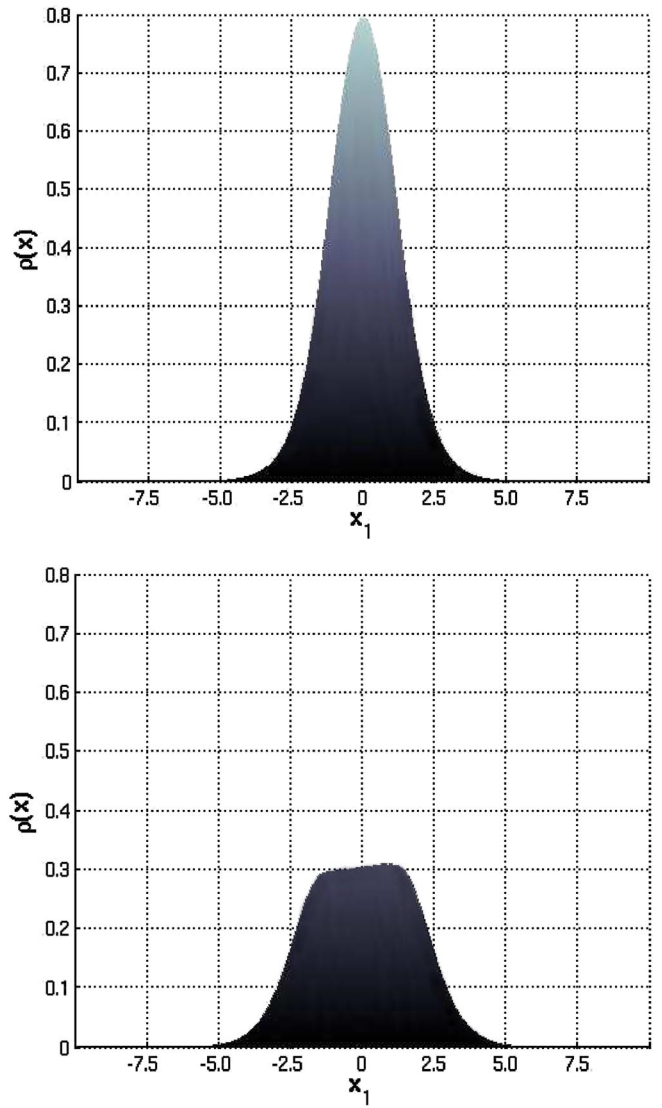


FIG. 2 (color online). Energy density  $\rho(\mathbf{x})$  of an oscillon in  $\phi^4$  theory at the moment of crossing the minimum of the potential (above) and at the moment of maximum excursion (below), corresponding the field  $\phi$  shown by the lowest solid line in Fig. 1.

time intervals. The length of the interval for the data shown is 5000 in natural units, which amounts just over 0.1% of the total length covered by the simulation. An interval of that length includes approximately  $10^3$  oscillations. A typical example of the power spectrum is shown in Fig. 6. There are very distinctive peaks that rise several orders of magnitude higher than the background between them. The first, highest peak just below frequency  $\omega = m$  indicates the oscillation frequency, the other peaks are located at integer multiples of that. Up to seven peaks can be identified in the power spectra.

There is a significant change in the shape of the peaks during the time evolution. In the beginning, when oscillon radiates and the core loses energy reasonably fast, peaks

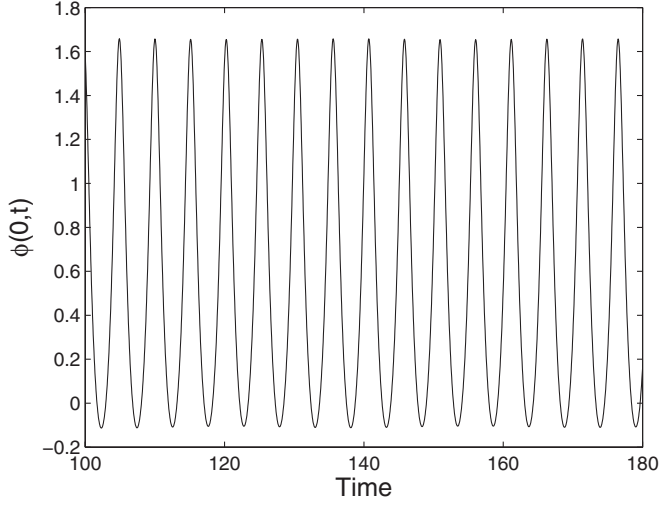


FIG. 3. Field at the center of the  $\phi^4$  oscillon  $\phi(t,0)$  as a function of time  $t$ .

are fairly broad, but later on when oscillon “stabilizes” they become extremely narrow. During the early evolution it is possible to make a fit to Breit-Wigner formula

$$\sigma(\omega) = \frac{K}{(\omega - \omega_0)^2 + (\Gamma/2)^2}, \quad (10)$$

where  $\omega_0$  is now the peak frequency. A fit to the first peak is shown in Fig. 7. It cannot be considered very accurate, but it gives an order of magnitude estimate of the decay width  $\Gamma$  during the era oscillon radiates its energy strongly. Later on the peaks have no width in the restricted time interval where the Fourier transformation is made (our

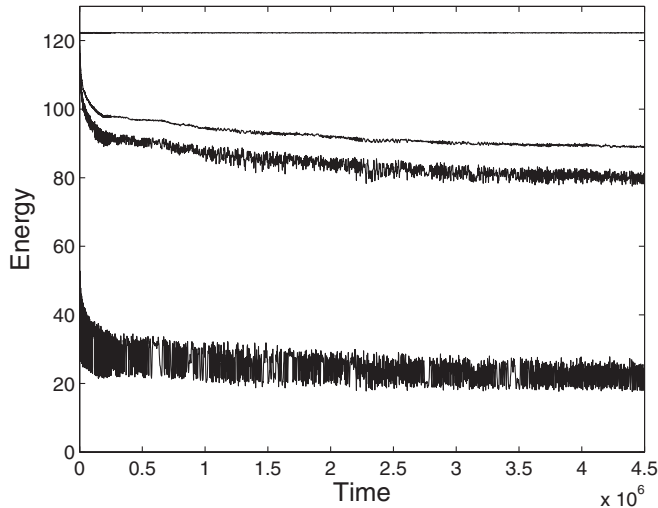


FIG. 4. Total energy in the lattice and energy inside the consecutive shells with radius  $r = 2.5r_0$ ,  $1.5r_0$  and  $0.5r_0$  around the center of the  $\phi^4$  oscillon (from top to bottom) as a function of time  $t$ .

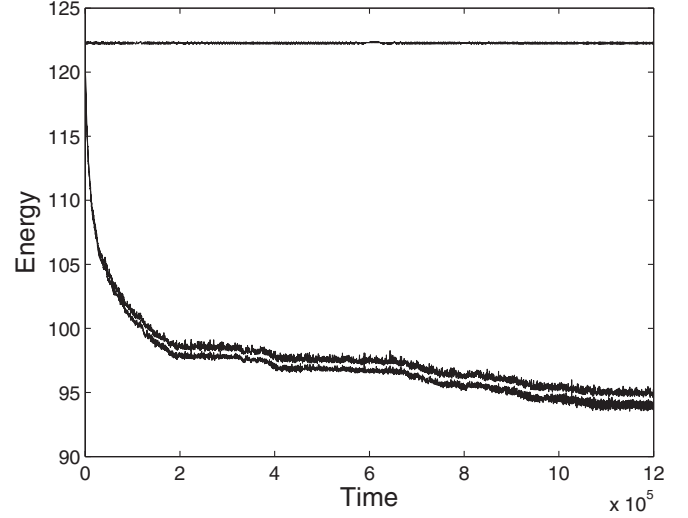


FIG. 5. Detail from Fig. 4, showing the total energy in the lattice and the energy inside shells with radius  $r = 5r_0$  and  $2.5r_0$  around the center of the  $\phi^4$  oscillon (from top to bottom).

choice for the length of the interval is fairly optimal as longer intervals tend to reveal a shift of the peak frequency, not improve the width).

The amplitudes of the peaks in Fig. 6 obey clearly exponential decay law as a function of the frequency  $\omega$ . The deviations from that at the level shown in the figure are most likely once again due to the limited accuracy of discrete Fourier transformation. The data also points out a low frequency peak (just next to  $|\phi(\omega, 0)|$  axis), with

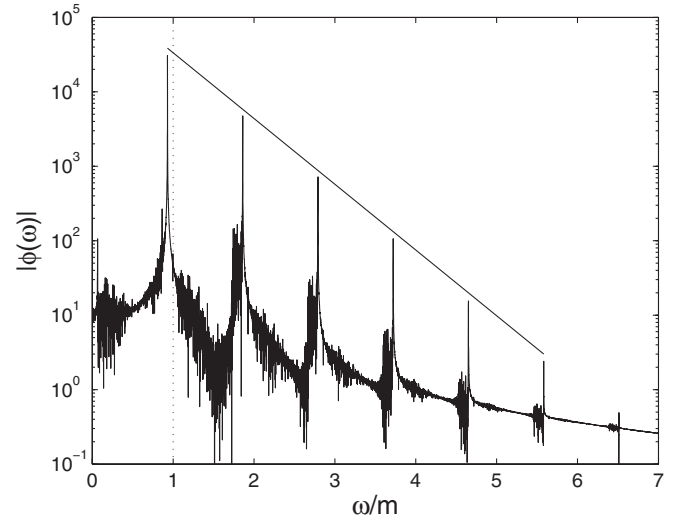


FIG. 6. Power spectrum of field at center of  $\phi^4$  oscillon  $|\phi(\omega)|$  taken in time interval  $2.5 \cdot 10^5 < t < 2.55 \cdot 10^5$ . Solid line is a guide to eye for an exponential fit to the amplitudes of the first six peaks,  $\exp(-a\omega/m)$  with a slope  $a = 2.03$ . Vertical dotted line shows the radiation frequency  $\omega = m$ .



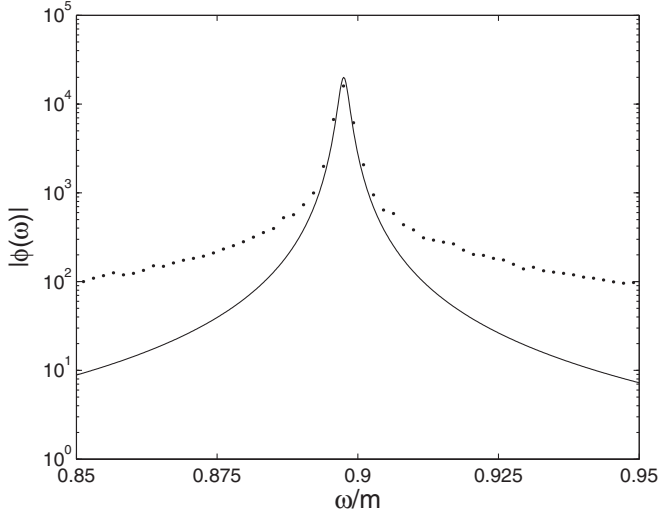


FIG. 7. Fit (solid line) to Breit-Wigner form of first peak (dots)  $\phi^4$  oscillon taken in time interval  $10^4 < t < 1.25 \cdot 10^4$ :  $\omega_0 = 0.8975$ ,  $\Gamma = 0.0020$ , and  $K = 0.02$ .

frequency approximately 0.06. There is a slight variation of the amplitude, a beat, with the corresponding period around 100 in time units (though not present in Fig. 3), which we believe is the cause of the structure.

Not only the shape of the peaks but also their location changes during the time evolution. There exists a strong correlation between the energy in the oscillon (shell radius  $r = 2.5r_0$  in Fig. 4) and the oscillation frequency, i.e. location of the highest peak, which is shown in Fig. 8 as a function of time. During the early period when the

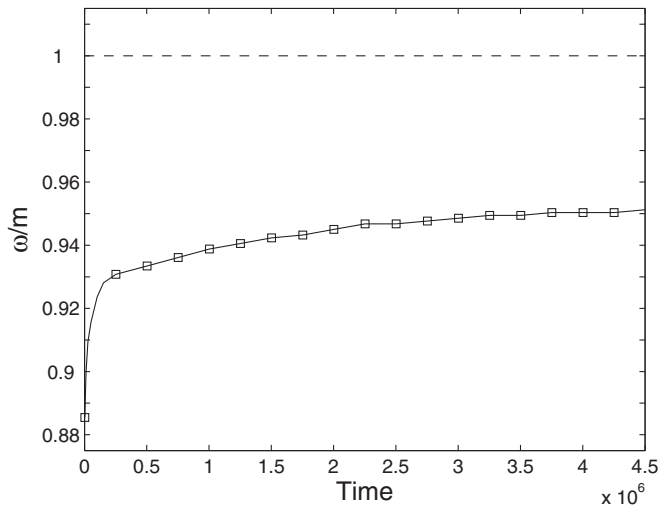


FIG. 8. Oscillation frequency corresponding to the location of the highest peak as a function of time for  $\phi^4$  oscillon. The precision of the measurement of the frequency is 0.002 at the scale of  $\omega/m$  due to the limited time interval the Fourier transformation is made. Some data points close to the frequency axis are not marked with  $\square$  for clarity. Dashed line shows the radiation frequency  $\omega = m$ .

oscillon radiates strongly, the frequency increases rapidly, but then the growth slows down drastically as the rate of energy loss becomes tiny. The time evolution of the oscillation frequency is the key to predict the life time of an oscillon: as long as the frequency stays below radiation frequency, oscillon cannot directly radiate all of its energy and disappear. As the rate of energy loss decreases, the increase in frequency slows down. Turning it other way round this is in agreement with Watkins [12] who reported the radiation rate decrease as  $\omega/m$  increases, with a minimum at  $\omega/m \approx 0.97$ .

Though we have not evolved oscillons in our simulation longer than  $4.5 \times 10^6$  time units, we can give bounds on the life time of oscillon on the basis of the evolution of the oscillation frequency. Even a linear fit to second half of the points in Fig. 8 suggests that the radiation frequency is not reached before a time of a few times  $10^7$ , or about  $10^7$  oscillations. Similar lifetimes were reported in [24] where lattices with a technique of adiabatic damping were exploited. However, the life time can be much longer as the slope of the line in Fig. 8 is clearly flattening out as well as the strongly decreasing radiation rate as a function of frequency reported in [12].

Setting the original amplitude  $A$  too high (e.g.  $A = 10$  to the direction where the potential is steeper) will not lead to an oscillon, the initial concentration of energy does not stay localized but spreads rapidly. Starting with a configuration that has more energy than the one shown in Fig. 1 can still evolve to an oscillon but with a modulated amplitude. This effect becomes most apparent in the case of sine-Gordon potential.

## B. Sine-Gordon potential

Persistent oscillations in radially symmetric sine-Gordon equation were studied in [20] and extreme sensitivity both to the form and the amplitude of the initial deviation from the vacuum were observed. A complicated initial profile was used to obtain a minimally radiating pseudobreather. We have not been able to create a “minimal” oscillon with a Gaussian initial ansatz—though the profile of an oscillon in sine-Gordon potential is also fairly close to Gaussian (Fig. 9) oscillations have modulated amplitude (Fig. 10) the period of which depends on the initially set amplitude  $A$ . Figure 11 shows total energy in the lattice and energy inside shells of several radii around the center of the oscillon in a simulation that spans the evolution up to time  $10^6$ . There is no apparent evidence for energy plateaus in Fig. 11.

The same choice of the interval for Fourier transformation was used as in  $\phi^4$  theory that now corresponds 0.5% of the total time of the simulation. The study in frequency space shows similar basic features as in  $\phi^4$  theory with some significant differences. The sine-Gordon potential (4) is symmetric around minima in contrast to quartic potential (3). Because of this symmetry there are no even harmonic

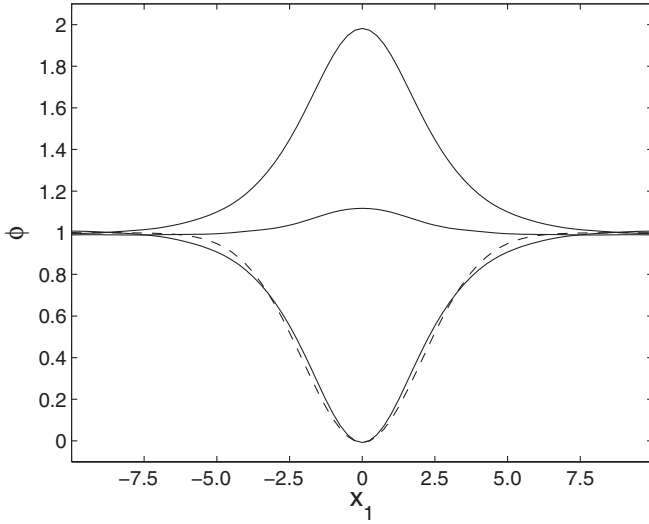


FIG. 9. Oscillon profile at the extrema (above and below the vacuum expectation value) and at crossing the minimum of the potential in the 2D sine-Gordon theory. Dashed line shows a Gaussian form with the same amplitude and the width  $r_0$  of the initial profile.

peaks in the power spectrum of sine-Gordon oscillon as can be seen in Fig. 12 and at very best only first four peaks were visible in the power spectrum. As the amplitudes of the peaks decrease exponentially also here, but only half of them are present compared to Fig. 6 of  $\phi^4$  theory, this suggests that the ansatz (9) would be a particularly good approximation for an oscillon in sine-Gordon potential, or potentially in any other symmetric potential that allows oscillons. There is no sign of low frequency structure in Fig. 12 in contrast to Fig. 6.

The time evolution of the frequency of the first peak in power spectrum of sine-Gordon oscillon is shown in Fig. 13 from a simulation that was evolved  $10^6$  time units.

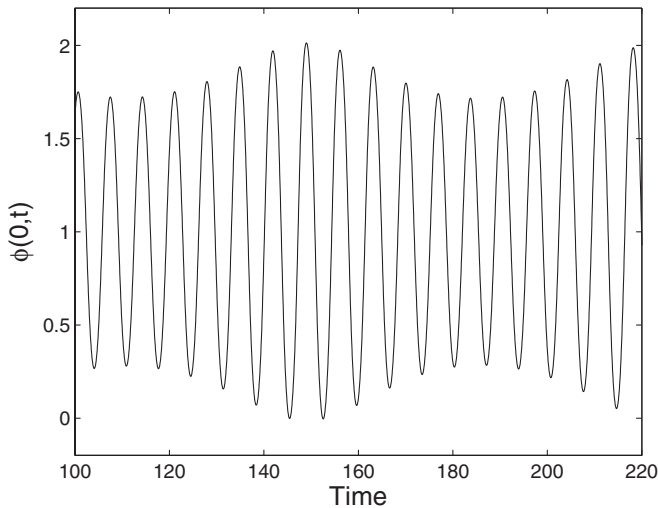


FIG. 10. Field at the center of the sine-Gordon oscillon ( $\phi(t, 0)$ ) as a function of time  $t$ .

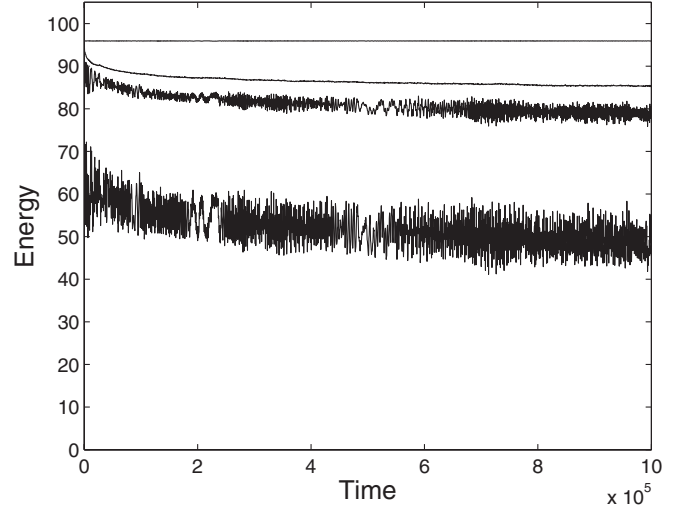


FIG. 11. Total energy in the lattice and energy inside the consecutive shells with radius  $r = 5.0r_0$ ,  $2.0r_0$  and  $1.0r_0$  around the center of the sine-Gordon oscillon (from top to bottom) as a function of time  $t$ .

The oscillation frequency starts initially closer to the lowest radiation frequency compared to the quartic potential, but there is no drastic increase either, which may be due to the weaker coupling to radiative modes as even multiples of the oscillation frequency are absent. Also here the increase in the oscillation frequency slows down in the course of time and even linear extrapolation yields a lifetime estimate, i.e. intersection with the radiation frequency, around  $10^7$  time units, but it could be much larger, especially as the study of the power spectrum suggests weaker radiative modes.

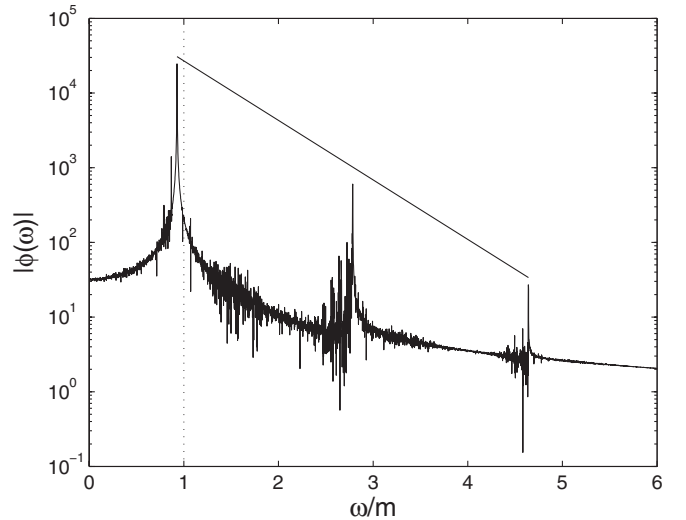


FIG. 12. Power spectrum of field at center of sine-Gordon oscillon  $|\phi(\omega)|$  taken in time interval  $1.5 \cdot 10^5 < t < 1.55 \cdot 10^5$ . Solid line is guide to eye for an exponential fit to peak amplitudes,  $\exp(-a\omega/m)$  with a slope  $a = 1.84$ . Vertical dotted line shows the radiation frequency  $\omega = m$ .

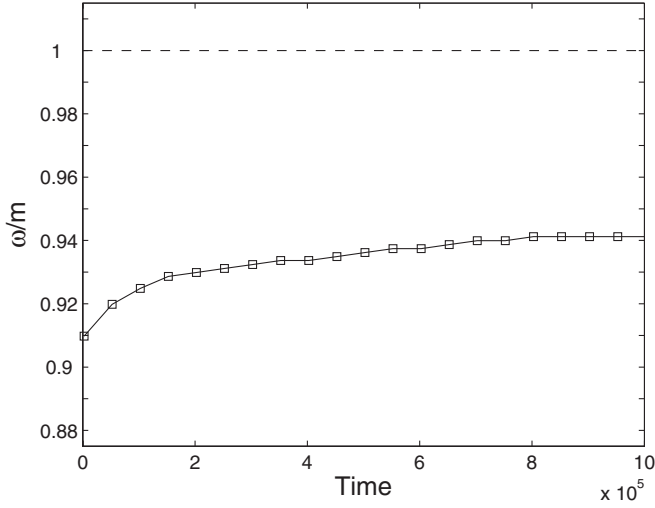


FIG. 13. Oscillation frequency as a function of time for sine-Gordon oscillon. The precision of the measurement of the frequency is 0.003 at the scale of  $\omega/m$ . Dashed line shows the radiation frequency  $\omega = m$ .

#### IV. ELLIPTICAL OSCILLONS

The relative ease to create an oscillon together with the energy shells pointing to the existence of successive plateaus and thus a series of metastable states gives rise to the question if the field configuration of an oscillon is unique. A very modest approach to the uniqueness is a study of an initial configuration which is not spherically symmetric. As our numerical method was not restricted to spherical symmetry a Gaussian ansatz with different width in  $x_1$  and  $x_2$  direction was used. Oscillons emerging from the collapse of asymmetric bubbles was studied in [25] using effective radius as a measure. We study here the time

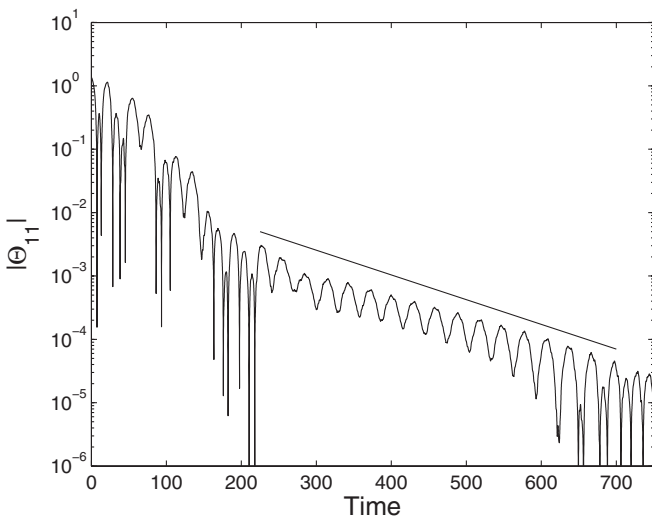


FIG. 14. Ellipticity,  $|\Theta_{11}|$ , as a function of time in the 2D  $\phi^4$  theory. Straight, solid line is guide to eye of an exponential fit,  $\exp(-bt)$  with a slope  $b = 0.009$ .

evolution of the ellipticity via the quadrupole moment  $\varrho_{ij}$  of the energy density  $\rho(t, \mathbf{x})$

$$\varrho_{ij} = \frac{\int d^2x x_i x_j \rho(t, \mathbf{x})}{\int d^2x \rho(t, \mathbf{x})}. \quad (11)$$

Ellipticity is given by the eigenvalues of the traceless matrix

$$\Theta_{ij} = \varrho_{ij} - \frac{1}{2} \delta_{ij} \varrho_{ij}. \quad (12)$$

Because the major and minor axis are along  $x_1$  and  $x_2$  the off-diagonal entries  $\Theta_{12}$ ,  $\Theta_{21}$  vanish (a good check for the numerical accuracy of the method:  $|\Theta_{12}|, |\Theta_{21}| < 10^{-15}$ ) and then ellipticity is given directly by the diagonal elements of (12). We measured  $\varrho_{ij}$  in a square of length 20 in physical units located in the center of the lattice. Lattice size was set to be  $4800^2$  and thus no boundary effects will

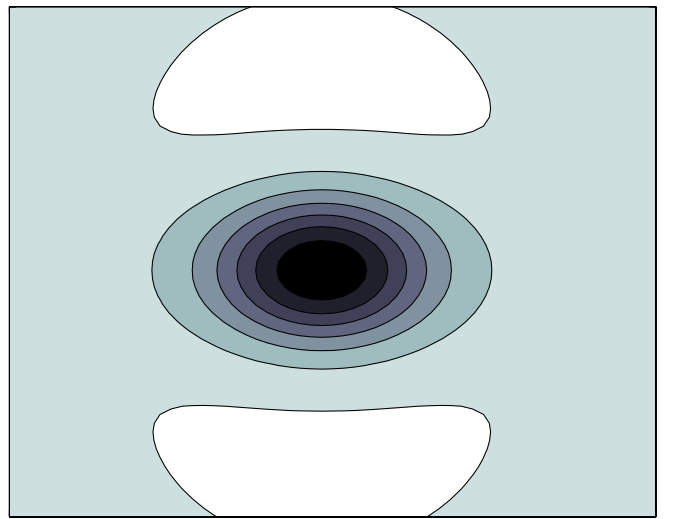
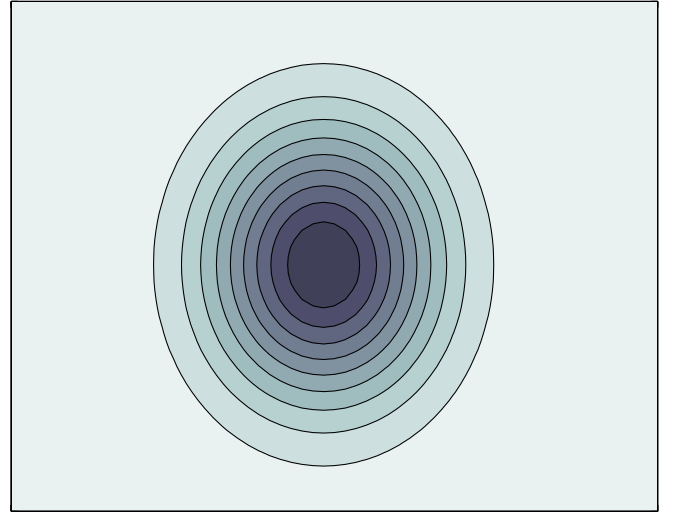


FIG. 15 (color online). Contours of  $\phi(\mathbf{x})$  for the initial elliptic Gaussian profile and after two oscillations (time = 10.6) in the 2D  $\phi^4$  theory.



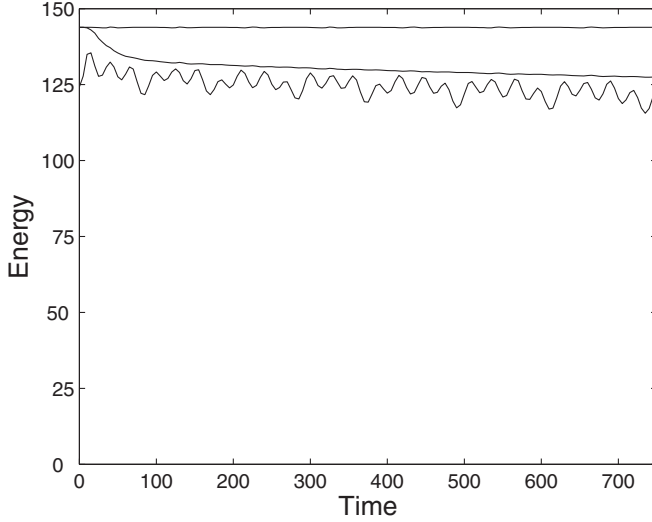


FIG. 16. Total energy and energy inside shells of radius  $r = 2.5r_0$  and  $r = 1.0r_0$  around the center of  $\phi^4$  oscillon as a function of time (from top to bottom). Here  $r_0 = 3.9$  is the initial width of the major axis.

be important until far after time = 600. Figure 14 shows  $|\Theta_{11}|$  as a function of time when initially the ratio of the major to the minor axes was set to be 3:2 (with the width in the  $x_1$  direction being  $r_0 = 3.9$ ). The oscillon approaches rapidly a spherical profile and ellipticity disappears in exponential phases. The orientation of the major axis changes in a way that is not obviously periodic (zero crossings of  $\Theta_{11}$  appearing in the Fig. 14 as almost vertical lines, which is demonstrated much more illustratively in Fig. 15). Figure 16 shows total energy and energy inside two shells around the oscillon core. Though spherically symmetric shells are not an entirely adequate method to measure energy of an elliptic configuration, they give a

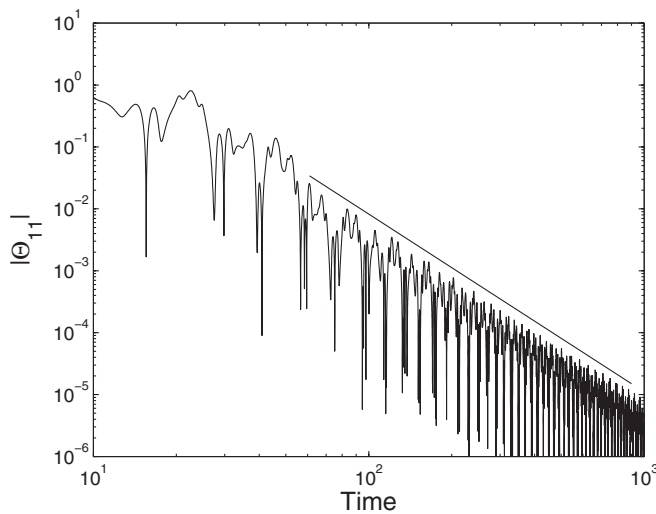


FIG. 17. Ellipticity,  $|\Theta_{11}|$ , as a function of time in the 2D sine-Gordon model. Straight, solid line is guide to eye of a power law fit,  $t^{-\delta}$  with a slope  $\delta = 2.87$ .

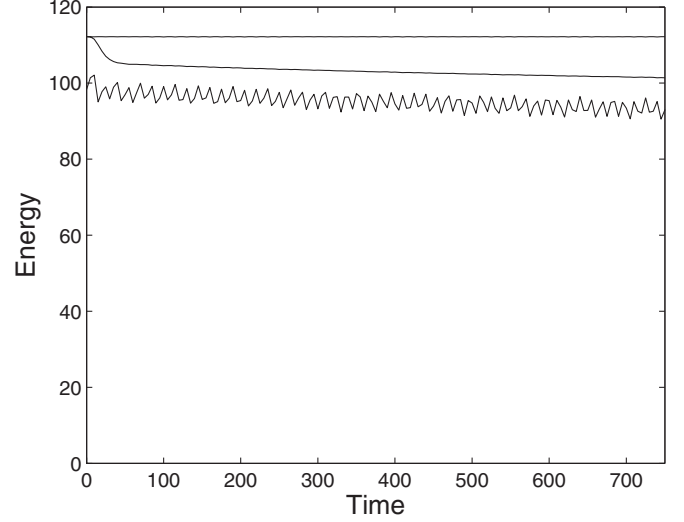


FIG. 18. Total energy and energy inside shells of radius  $r = 2.5r_0$  and  $r = 1.0r_0$  around the center of sine-Gordon oscillon as a function of time (from top to bottom). Here  $r_0 = 3.9$  is the initial width of the major axis.

reasonable estimate and show that while ellipticity disappears exponentially decreasing 4 orders of magnitude, the energy inside the oscillon core has decreased by just over 10% indicating a quick collapse to a spherical shape.

Figure 17 shows the ellipticity and 18 total energy and energy inside shells in a simulation with the same setup but for sine-Gordon potential. Qualitatively features are similar, ellipticity decays rapidly and oscillon collapses into spherical profile. Here the orientation of major axis changes frequently in almost periodic way. Moreover, the decay of ellipticity  $|\Theta_{11}|$  (in Fig. 17) matches better to a power law than an exponential fit. It is very tempting to try to interpret this result on the basis of the obtained differences of oscillons in frequency space between quartic and sine-Gordon potentials. As both even and odd multiples of the oscillation frequency are present in case of the quartic potential, the first radiative frequency, twice the basic frequency, has far greater amplitude than in sine-Gordon potential where the first radiative mode has stronger suppression its frequency being 3 times the basic oscillation frequency. Therefore it seems plausible that oscillon in quartic potential can radiate its asymmetry exponentially, while initial deviation from spherical symmetry decays only with a power law in sine-Gordon potential. Naturally, the study above does not exclude the possibility of different spherically symmetric forms for an oscillon.

## V. COLLIDING OSCILLONS

If created in Nature via some mechanism like collapse of domains, oscillons would have initially translational momentum. In order to draw the connection between oscillons given birth in simulations with random initial conditions and stationary oscillons created with e.g. Gaussian ansatz,

moving oscillons were prepared. Practically the oscillon that starts with a Gaussian initial profile is first allowed to evolve few oscillations and then the configuration is Lorentz boosted. For all the shown data the velocity of moving oscillons is set to be 0.5 (we point out that oscillons originating from collapsing domains seem to generally have higher velocities).

Fast moving oscillons become some sort of waves: they have typically at least two “phases,” part of the wave front lies above, part below the minimum of the potential. There is a precise correspondance between boosted oscillons and the objects travelling on wave fronts originating from collapsing domains. We have also checked that boosted oscillon is persistent like a stationary one and can circulate on a lattice (with periodic boundary conditions) travelling long distances without demise. Here it is appealing to speculate that there is a connection to freak waves in the context of pattern formation in oceanography (for a study of freak wave formation from a nonlinear Schrödinger equation see [26]).

We finish by reporting the results of studies where oscillons were made to collide. There is twofold reasoning for this study. We want to examine what kind of interactions oscillons have and thus try to understand their nature from that point of view. Secondly, if created in large numbers in phase transitions, collisions are inevitable and the behavior of oscillons in those has thus impact on the subsequent evolution of the system. The general rule is that a collision does not destroy an oscillon: we have never

witnessed that the first collision between oscillons would have lead to a demise of either of them though it is noticeable that oscillons disturbed this way have also disappeared in the second collision that occurred on a lattice with periodic boundary conditions.

Figure 19 shows snapshots from a collision between immobile and moving (velocity 0.5) oscillons. After the collision there is one fast moving oscillon continuing to the direction of original translational momentum and fairly stationary oscillon whose center, however, has moved slightly from the original location. This example seems thus almost interactionless. However, amount of interaction and momentum transfer depends on the phases of oscillons at the moment of collision and can result to two moving oscillons. This can be examined as the setup for collision has been prepared by creating the immobile oscillon at arbitrary time on the lattice, we can control the phase of stationary oscillon via the moment of its creation. The effect of the phase is demonstrated in Fig. 20 where the setup is otherwise the same as in Fig. 19, but the initially stationary oscillon is time 1.3 behind the previous one corresponding approximately  $1/4$  of the period. Here both the oscillons move left after the collision.

Figure 21 shows snapshots of a collision between two boosted oscillons with the same velocity (0.5) travelling into opposite directions. As can be seen oscillons pass through each other. We have also changed the alignment of oscillons so that the collision does not occur head on. Figure 22 shows snapshots where the deviation in the align-

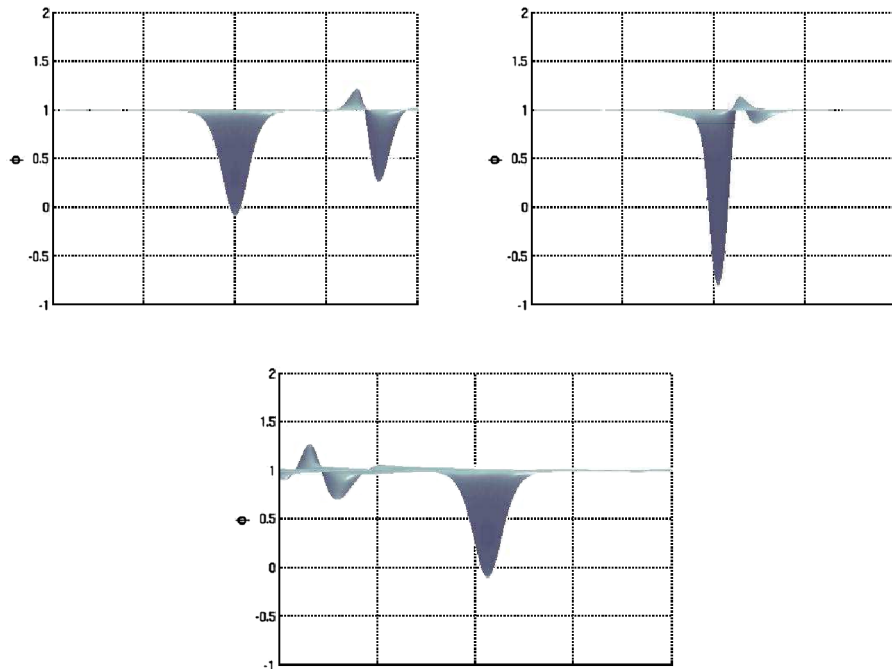


FIG. 19 (color online). Sequence of snapshots at times  $t = 0, 36.25, 80$  of an on-axis collision of two oscillons, an immobile (originally located in the middle) and moving (from right to left), in the 2D  $\phi^4$  theory.

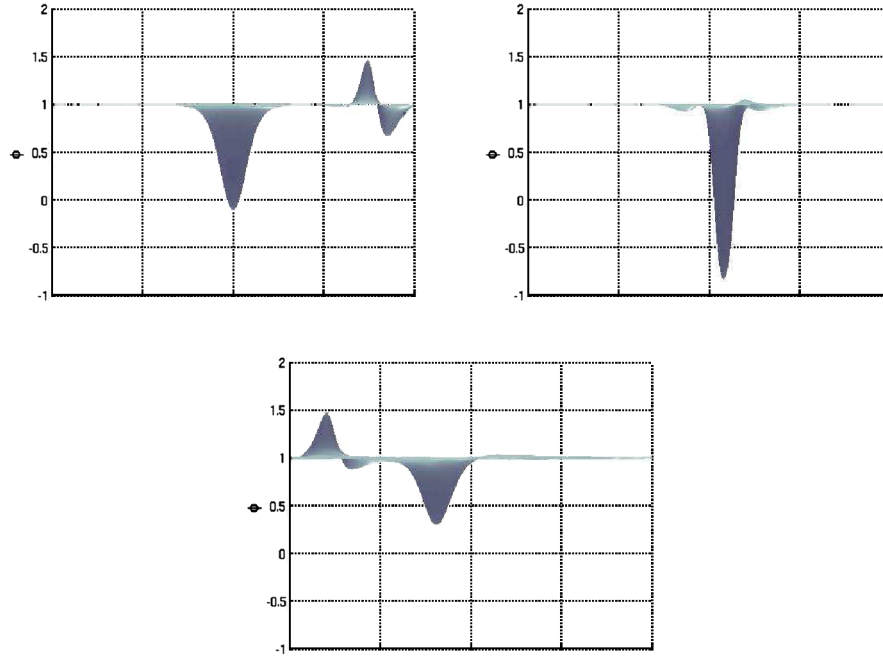


FIG. 20 (color online). Sequence of snapshots at times  $t = 0, 36.75, 100$  of an on-axis collision of two oscillons in the 2D  $\phi^4$  theory. The phase at collision set  $1/4$  of the period different compared to Fig. 19.

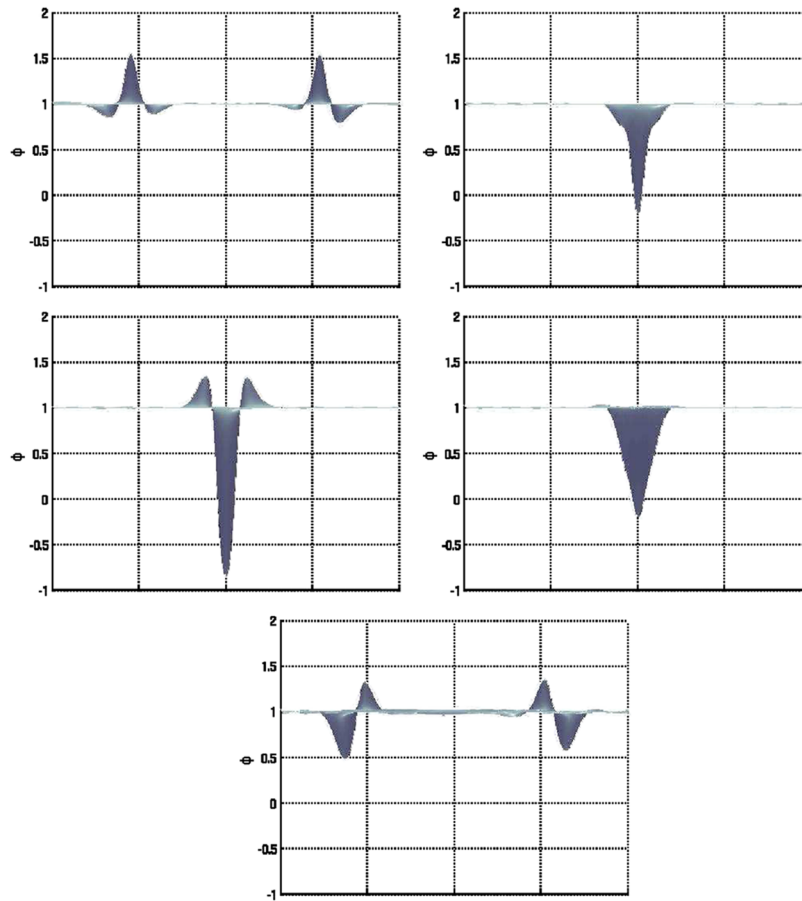


FIG. 21 (color online). Sequence of snapshots at times  $t = 0, 29.5, 30.5, 31.5, 60.5$  of an on-axis collision of two oscillons in the 2D  $\phi^4$  theory.

ment between the centers of oscillons is 5.0 in physical units. The setup leads to an attractive scattering, oscillons do not behave as classical particle like objects, but their paths bend towards each other the angle between initial and final velocity being approximately  $20^\circ$ .

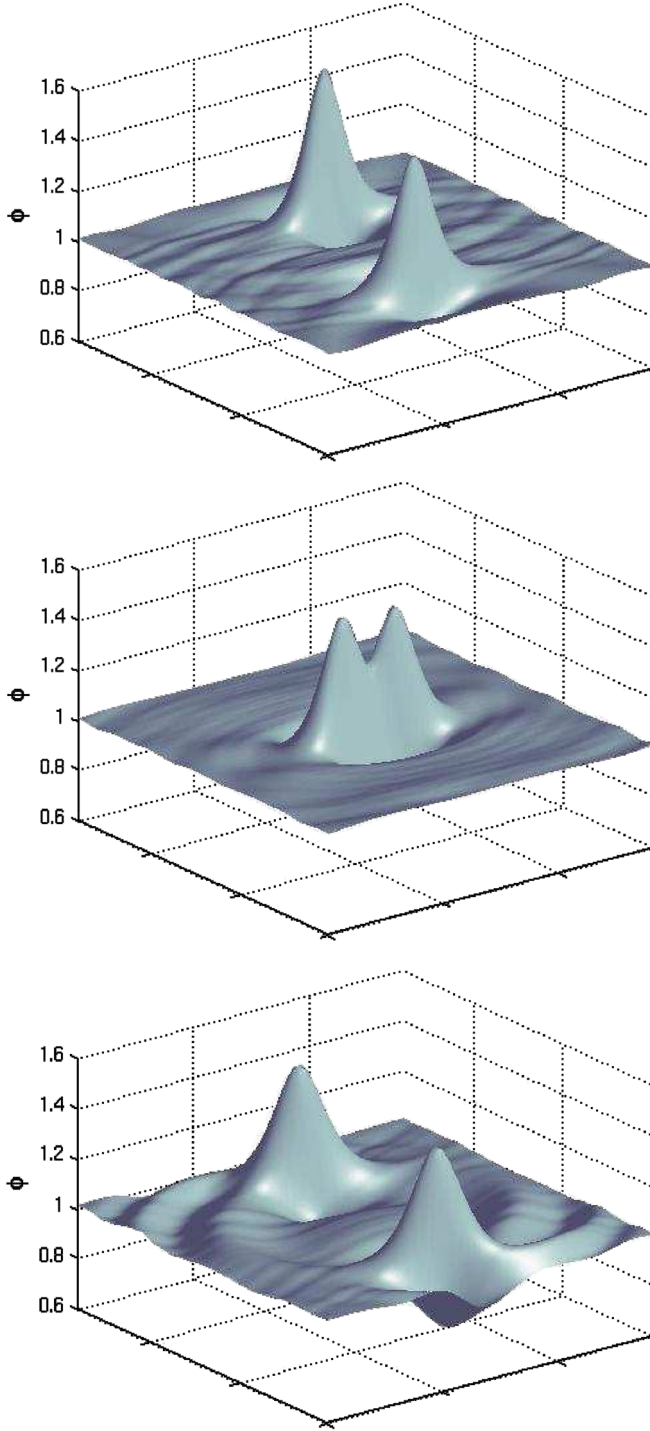


FIG. 22 (color online). Sequence of snapshots at times  $t = 0, 22, 49$  of an off-axis collision of two oscillons in the 2D  $\phi^4$  theory.

## VI. CONCLUSIONS

We have studied oscillons, extremely long-living oscillations of a scalar field, numerically in two dimensions for  $\phi^4$  and sine-Gordon potentials. We evolved stationary oscillon for  $10^6$  time units or longer (which is three orders magnitude larger than the reported life-time for I-balls,  $10^3 m^{-1}$ , in [11]) and examined the power spectrum of oscillations. Study in the frequency space not only suggested much longer life time on the basis of the time evolution of the basic oscillation frequency than we could directly probe, but also points out the validity of the separable ansatz (9). Though we have not the evolved oscillon in sine-Gordon potential as long, the analysis of the power spectrum suggests that it might be even more persistent than oscillons in  $\phi^4$  theory in spite of the modulated amplitude. Studies of Lorentz boosted oscillons and collisions between them show also persistence of these objects and point out the behavior that is very similar to solitons.

As our study was carried out on lattices with periodic boundary conditions, the radiation oscillons emit comes to some extent back to the center of the lattice where oscillon is located. This can be viewed as a small perturbation and long survival of oscillons in the setup as their persistence to radiation. On the other hand incoming radiation can also work otherwise pumping energy back into oscillon and thus extending the life-time [12]. Even in the latter case our study has relevance in situation where oscillon can absorb energy from environment, like a weak heat bath or a laboratory experiment in a closed system.

There are several unanswered question concerning oscillons and their longevity. The mechanism that makes some potentials able to confine energy over millions of oscillations should be understood together with the relation to the oscillation frequency. On the basis of the spreading of the energy density during the oscillation (see Fig. 2) one would naively assume that energy could escape in the form of radiation relatively quickly. The oscillation frequency also seem to be fined tuned just below the radiation frequency. This has been reported also for the SU(2) model oscillon [21] where the difference between oscillation and radiative frequencies was observed to be per mill level. Finally, even the level and role of nonlinearity in the nature of oscillons is not well understood. It has been noticed, both in the study of freak wave formation from a nonlinear Schrödinger equation [26] as well as oscillon (called axiton by the authors) formation in the axion field [27], that nonlinear effects are crucial in the generation of high density contrasts. On the other hand, in [11] the authors emphasize the importance of the potential being dominated by the quadratic term as a condition for I-balls to exist as well as in [12] weakness of nonlinear effects was considered important for the longevity of oscillons.

Almost stable, nondissipative solutions of field theories oscillons have interest itself, but also their existence can

lead to significant consequences that are worth studies of their own. In particular, if formed, oscillons could play a role in the early universe and phase transitions that occurred there especially if they survive long enough in realistic conditions, as for instance their behavior in collisions suggests. Here e.g. the oscillon in sine-Gordon potential may have relevance in the QCD phase transition as the axion potential has sine-Gordon form. Kolb and Tkachev have studied the formation of solitons in [27]. While finishing this paper we became aware of the study [28] where similar life times of oscillons as we obtained here were reported, but in an expanding background in one dimension with a potential including also  $\phi^6$ -term. If this longevity persists in full three dimensional models with Hubble expansion, oscillons may have interesting cosmological implications.

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*Note added.*—While this paper was being considered for publication the study [29] appeared examining oscillons in 3 dimensions.

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- [1] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974).
  - [2] A. M. Polyakov, JETP Lett. **20**, 194 (1974).
  - [3] S. R. Coleman, Nucl. Phys. **B262**, 263 (1985).
  - [4] M. K. Prasad and C. M. Sommerfield, Phys. Rev. Lett. **35**, 760 (1975).
  - [5] S. Theodorakis, Phys. Rev. D **61**, 047701 (2000).
  - [6] R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982).
  - [7] I. L. Bogolyubsky and V. G. Makhankov, JETP Lett. **24**, 12 (1976).
  - [8] I. L. Bogolyubsky and V. G. Makhankov, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 120 (1977).
  - [9] M. Gleiser, Phys. Rev. D **49**, 2978 (1994).
  - [10] E. J. Copeland, M. Gleiser, and H. R. Muller, Phys. Rev. D **52**, 1920 (1995).
  - [11] S. Kasuya, M. Kawasaki, and F. Takahashi, Phys. Lett. B **559**, 99 (2003).
  - [12] R. Watkins, DART-HEP-96/03, 1996 (unpublished).
  - [13] E. P. Honda and M. W. Choptuik, Phys. Rev. D **65**, 084037 (2002).
  - [14] M. Gleiser and R. C. Howell, cond-mat/0310157.
  - [15] I. Dymnikova, L. Koziel, M. Khlopov, and S. Rubin, Gravitation Cosmol. **6**, 311 (2000).
  - [16] A. Riotto, Phys. Lett. B **365**, 64 (1996).
  - [17] M. Gleiser, hep-th/0602187.
  - [18] M. Gleiser and R. M. Haas, Phys. Rev. D **54**, 1626 (1996).
  - [19] M. Gleiser, Phys. Lett. B **600**, 126 (2004).
  - [20] B. Piette and W. J. Zakrzewski, Nonlinearity **11**, 1103 (1998).
  - [21] E. Farhi, N. Graham, V. Khemani, R. Markov, and R. Rosales, Phys. Rev. D **72**, 101701 (2005).
  - [22] M. Broadhead and J. McDonald, Phys. Rev. D **72**, 043519 (2005).
  - [23] E. J. Copeland, S. Pascoli, and A. Rajantie, Phys. Rev. D **65**, 103517 (2002).
  - [24] M. Gleiser and A. Sornborger, Phys. Rev. E **62**, 1368 (2000).
  - [25] A. B. Adib, M. Gleiser, and C. A. S. Almeida, Phys. Rev. D **66**, 085011 (2002).
  - [26] M. Onorato, A. R. Osborne, M. Serio, and S. Bertone, Phys. Rev. Lett. **86**, 5831 (2001).
  - [27] E. W. Kolb and I. I. Tkachev, Phys. Rev. D **49**, 5040 (1994).
  - [28] N. Graham and N. Stamatopoulos, Phys. Lett. B **639**, 541 (2006).
  - [29] G. Fodor, P. Forgacs, P. Grandclement, and I. Racz, hep-th/0609023.